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PLANNING OF CUTTER PATH IN HIGH SPEED MACHINING BASED ON MAPPING OF TRIANGULAR GRID SURFACE

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Abstract- This paper puts forward an algorithm of generating a constant CNC spiral path based on curved surface of triangular grid. This algorithm takes fully consideration of the geometry and dynamics characteristics of the cutter-path and triangular grid cell, effectively reducing numbers of cutter lifting and maintaining the continuity of the cutter-path. This algorithm starting biasing from the contact points of cutter on the boundary of profile curve of grid layer by layer and planning the contact points of biased cutter on the parametric field, fitting contact points onto the spiral path by NURBS fitting algorithm, finally the spiral cutter-path is obtained by inverse mapping it on the grid. The algorithm in this paper can generate a path with advantages of consistence boundary, smooth and continuous path and without repeatedly need for feeding or relieving of cutter compared with those paths generated through traditional ways.

Key words: Triangular, Grid cell, Mapping rule, Spiral cutter-path

I. INTRODUCTION

High-speed NC machining technology has taken larger and larger proportion in the modern machining and manufacturing technology with its characteristics of high efficiency and high surface quality etc. It also has obvious advantages in the machining of complex free surface and has become the main approach for machining of curved surface. High speed machining not only has to consider the problem of machine tool, fixtures and cutters, the dynamic performance of machine tools and planning ways of machining path is taken into account at the same time, the discontinuous path or mutations of path will inevitably lift the cutter that may cause the vibration and impact to the machine tool. Thus to ensure the uniformity of cutting load and fewer changes of velocity direction to the greatest extent in the process of high speed machining is of great importance, when planning the cutter-path, try to set smooth transition for movements for feeding or relieving of cutter, interval and non-cutting movements, set transition connection with circular arc in the place of cutter-path that has big change of curvature. Zuqiang Long, Yan Yuan etc [1] [2] proposing design fuzzy controllers based on variable universe of discourse and using it in high-accuracy positioning of lathe servo system. With the rapid development of CAD/CAM technology, especially with the rapid development of 3d printing in recent years, STL triangular grid is widely used in modeling of discrete geometry modeling, NC machining programming, graphics and images etc. Compared to the parametric geometric model, triangular grid model is linear approximating to the original model in the form of triangle surface patch, thus there necessary being the precision loss of model expression. Current ways for machining grid model is usually using the cross section ways [3] , parameter ways [4] and residual height ways [5] [6] to generate machining cutter-path. Using these ways to generate cutter-path, due to the cutter-path is inconsistent with the boundary of model, often there will generate segmented machining path that will cause continuous cutter lifting during machining, it may cause vibration and impact to the machine tool, fluctuation of load and increasing wear of cutters, which will seriously affect the machining efficiency and machining precision. The best way for machining grid model is using spiral feeding to plan the machining path. This way that is using ball-end-cutter machining that can make machining path continuous, reduce the number of cutter lifting and guarantee the machining quality of products, so choosing this way of feeding is very promising.

Currently in the study of CNC path planning, the studies of machining path planning ways for parametric curved surface by using spiral cutter have been profound made. Because of the problems of precision loss in the triangular grid curved surface, the way of path planning is quite complicated and the machining precision of obtained spiral path cannot be guaranteed, so there are fewer study of this aspect relatively. ZHU etc [7] proposed a generation way for spiral path by using consistent residual height. LEE [8] extracted the boundary curve, using consistent residual height to get the distance of adjacent path and bias boundary curve to obtain a bunch of annular path. Xu Jinting etc [9] obtained the machining path through biased the contour curve. The ways mentioned above cannot obtain the smooth machining path and are not suitable for high speed machining. ZHOU etc [10] proposed a spiral machining path based on the parametric curved surface, the connection of adjacent path was using the mapping points that cannot get smooth path. PATELOUP etc [11] biased the external contour of model, using interpolation of B-spline curve around the corner to realize the smooth connection of path around the corner, but the machining path in actual process wasn't continuous and smooth. LI M etc [12] developed a fast recursive search way. LU B etc [13] proposed a planning way of spiral path based on contours and consistent residual height. The four ways mentioned above are mainly obtaining the spiral machining path indirectly quoted to other models and the operation are complex, they are not take advantages of the characteristics of the triangular grid curved surface.

There is still lack of a planning spiral path algorithm for triangular grid by using its own characteristics.

In this paper, through extracting the boundary curve on model of triangular grid surface, the distance of adjacent contact points of cutter path is determined according to consistent residual height and curvature model of curved surface, series clusters of annular contact points of cutter are obtained by biasing the contact points of cutter on the basis of dispersing the boundary curve into contact points of cutter, specifying the rules of mapping parameters and boundary conditions, mapping the contact points of cutter in parametric field according to the certain rules of mapping, re-planning the contact points of cutter and the contact points of cutter in mapping parametric field is obtained, the spiral curve is obtained by using NURBS interpolation algorithm to interpolate contact points of cutter, inverse mapping the spiral curve on the curved surface of grid to obtain the spiral machining path. According to the simulation of proposed algorithm, the results show that this algorithm can obtain a smooth spiral machining path with small changes of curvature and consistent residual height.

In this paper, the flow chart of algorithm is shown in figure 1. the row pitch is determined by consistent residual height and curvature of curved surface of grid, step length of feeding is controlled by the error of arc height, among them: (1) Boundary curve is the largest contour curve of the grid model; (2) The contact points of cutter on curved surface of triangular grid is determined by residual height, curvature of cutter contact points on the curved surface of grid and error of arc height; (3) The initial biggest contour curve is dispersing into the contact points of cutter according to error of arc height; (4) Establishing a field of mapping parameters, specifying mapping rules and re-planning the cutter contact points in parameter mapping field, using fitting algorithm of NURBS curve to fit the contact points of cutter onto spiral path in parameter filed; (5) The spiral machining path is obtained by inverse mapping the re-planned spiral path on curved surface of grid.

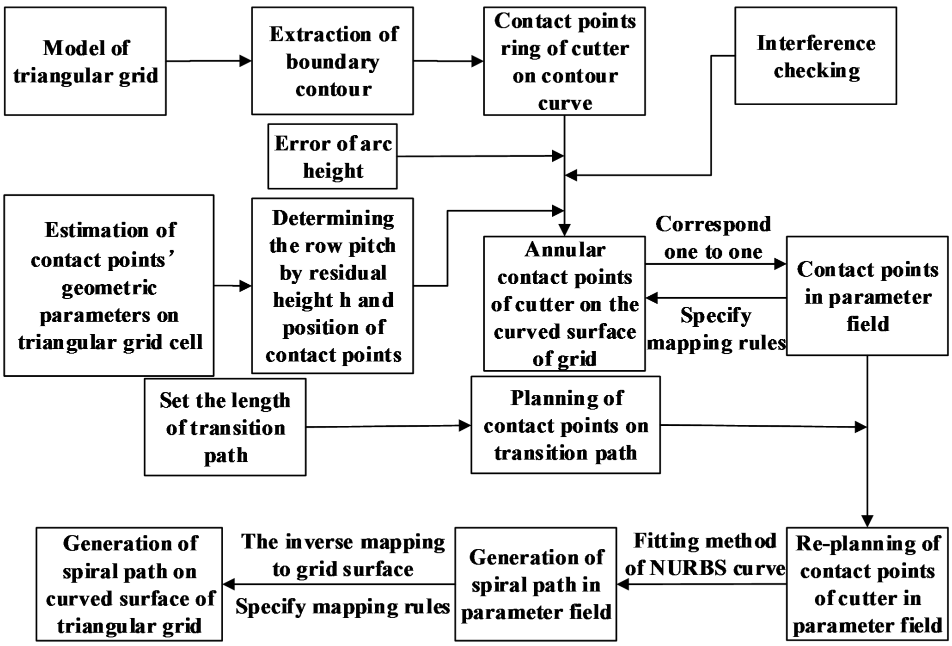


Figure 1. Flow chart of algorithm

II. CHARACTERISTIC ANALYSIS FOR CURVED SURFACE OF TRANGULAR GRID

Before the work of path planning, analysis for geometric characteristics of contact points of cutter on curved surface of triangular grid, mainly through the description of normal vectors and the curvature, this work provides basis for the follow-up work which is planning the path of cutter to go on smoothly. In order to facilitate the follow-up call of normal vectors and the curvature of pitch points, the normal vectors and the curvature of pitch points which are on the curved surface of triangular grid are calculated and stored in a file, it is convenient for the use for follow-up work. This paper is based on grid model format of STL, using the topology of the triangular grid file, traversing the grid and solving the grid cell by using the corresponding way of solution, according to solution of the parameters to find the corresponding number of triangular grid cells that meet the boundary conditions and further analysis of relationships between the position of cutter contact points and the triangular grid model is done. Use the positive and negative relationships between contact points of cutter and edge of grid cell to determine the position relationships of contact points of cutter and the curved surface of triangular grid cell. The relationships are divided into three kinds that are: the contact points of cutter which are on the vertex of grid cell, the contact points of cutter which are on the edge of grid cell and the contact points of cutter are inside the grid cell. The contact points of cutter $P(u, v)$ are obtained by computing and providing the original data for follow-up mapping.

A sketch of grid cell is shown in figure 2, P_1, P_2 and P_3 are vertexes of triangle grid cell, P' is the contact point of cutter, its expression is: $P'=P_1+u(P_2-P_1)+v(P_3-P_1)$. The boundary conditions of position relationships between P' and the grid cell are as follows: when $u=0$ and $v=0$, P' is at the position of P_1, when $u=1$ and $v=0$, P' is at the position of P_2, when $u=0$ and $v=1$, P' is at the position of P_3, when $0<u$, $v<1$ and $0<u+v<1$, P' is inside the grid cell, when any parameter of u and v is equal to 0, the other is less than 1 and greater than 0, P' is on the edge of the grid cell, when $u+v>1$, P' is not on the grid cell. The solution expression of u and v are show as follows:

$$u = \frac{((\mathbf{V}_1 \cdot \mathbf{V}_2)(\mathbf{V}_2 \cdot \mathbf{V}_0) - (\mathbf{V}_1 \cdot \mathbf{V}_0)(\mathbf{V}_2 \cdot \mathbf{V}_1))}{((\mathbf{V}_0 \cdot \mathbf{V}_0)(\mathbf{V}_1 \cdot \mathbf{V}_1) - (\mathbf{V}_0 \cdot \mathbf{V}_1)(\mathbf{V}_1 \cdot \mathbf{V}_0))}$$
$$v = \frac{((\mathbf{V}_0 \cdot \mathbf{V}_0)(\mathbf{V}_2 \cdot \mathbf{V}_1) - (\mathbf{V}_0 \cdot \mathbf{V}_1)(\mathbf{V}_2 \cdot \mathbf{V}_0))}{((\mathbf{V}_0 \cdot \mathbf{V}_0)(\mathbf{V}_1 \cdot \mathbf{V}_1) - (\mathbf{V}_0 \cdot \mathbf{V}_1)(\mathbf{V}_1 \cdot \mathbf{V}_0))} \quad (1)$$

In the expressions, $\boldsymbol{V_0}=P_2-P_1$, $\boldsymbol{V_1}=P_3-P_1$.

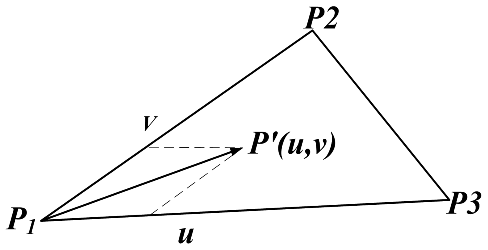


Figure 2. Diagram of coordinate expression of arbitrary contact point of cutter

The normal vectors of contact points of cutter on the curved surface are very important to determine the step length and row pitch, they are related to planning rationality of the cutter-path and has influence on machining precision of products as well. When the contact points of cutter are on the vertexes of grid cell, as are shown in figure 3, at this time, the vectors of contact points of cutter can be calculated by using algorithm of shape-area weight [14]. So the normal vectors of contact points of cutter that are on vertexes of grid cell can be expressed by:

$$\boldsymbol{n_i} = \frac{\sum_{k=1}^{m} \lambda_i S_i n_{i,k}}{\| \sum_{k=1}^{m} \lambda_i S_i n_{i,k} \|} \quad (2)$$

In the expression: λ is the weight of shape-are, S is the are of grid cell, $n_{i,k}$ are the normal vectors of grid cell.

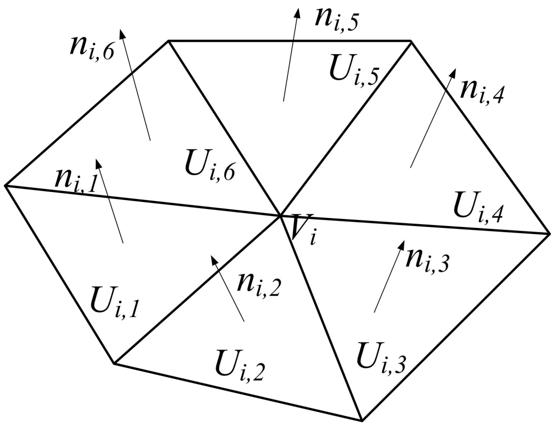


Figure 3. Contact points of cutters which are on the vertexes of grid cell

When the contact points of cutter are on the edge of grid cell, as are shown in figure 4, assuming that the contact point of cutter P_k is on the line V_mV_n, the normal vector of the contact point of cutter on the edge of grid cell is calculated by using the algorithm of weighted compensation of vector [15] n_k. The expression of calculation is:

$$n_k = (1-\delta)n_m + un_n + \beta(t) n_{comp}, 0 \le \delta \le 1 \tag{3}$$

In the expression: $n = \dfrac{(s_p n_p + s_q n_q)}{\sum s_{m,n}}$ n_m, n_n, n_p and n_q are the normal vectors of vertexes of pitch points of grid cell, S_m, S_n, S_p and S_q are the area of grid cell, $\beta(t) = t(1-t),\ t \in [0\ 1]$ is the The gradient influence function of compensation vector n_{comp}.

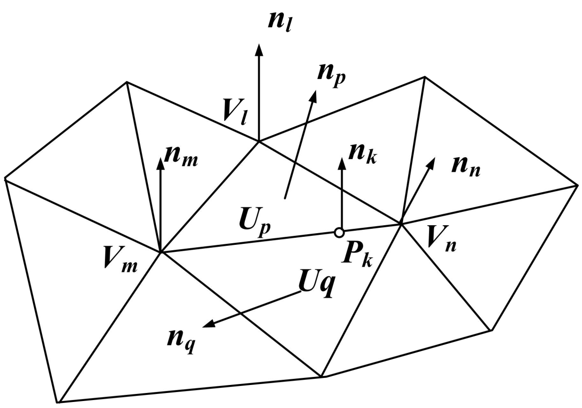


Figure 4. Contact points of cutter which are on the edge of grid cell

When the contact points of cutter are inside the grid cell, as are shown in figure 5, the normal vectors of contact points of cutter are calculated through the algorithm of quadratic linear weight on the normal vectors of grid cell. The expression of calculation is:

$$n_k = v[un_n + (1-u)\, n_l\,] + (1-v)\left[un_n + (1-u)\, n_m\,\right] \tag{4}$$

In the expression: n_l, n_m and n_n are normal vectors on the vertexes of grid cell.

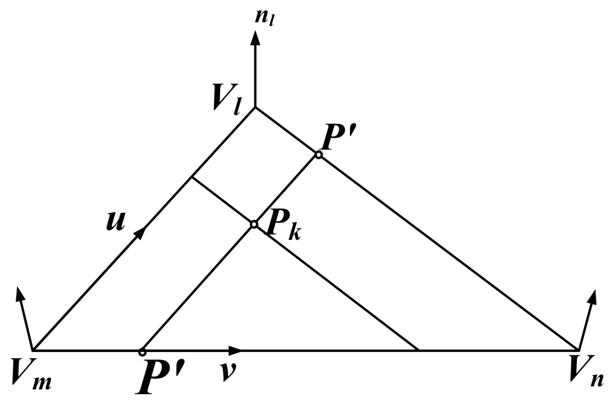


Figure 5. Contact points of cutter which is inside the grid cell

After the calculation of normal vectors of contact points of cutter on the grid cell, due to the curvature of current contact point of cutter has influence on determination of adjacent contact point of cutter, therefore, it is still need to calculate the curvature of contact point of cutter on the curved surface to provide basis for the selection of cutter, adjustment of machining position, convexity or concavity judgment of curved surface and interference check of cutter. In this paper, using the algorithm of least squares fitting to estimate the main curvature [16] of vertex on grid cell k_1, k_2, k_3 and main direction of vertex on grid cell e_1, e_2, e_3. According to Euler formula, the curvature of contact points of cutter on grid cell in arbitrary direction is calculated. The main expression of curvature is:

$$K = v[uk_2(\beta_2) + (1-u)\cdot k_3(\beta_3)\,] + (1-v)\,\left[u\cdot k_2(\beta_2) + (1-u)k_1(\beta_1)\right] \tag{5}$$

In the expression: k_1, k_2 are the main curvature of vertex P, e_1, e_2 are the main direction of of vertex P, β_i $(i=1,2,3)$ is the angle between the projection of arbitrary direction vector $\boldsymbol{d}$ which is translated on tangent plane of vertex and the normal vector of vertex, the relationship expression of normal curvature of contact point along arbitrary direction e' is:

$$k\left(e'\right)=k_1 \cdot cos^2\theta_1 + k_2 \cdot sin^2\theta_1 \qquad (6)$$

θ_1 and θ_2 are the angles between the arbitrary direction e' and main direction e_1 and e_2. The other relationships of position are shown in figure 6.

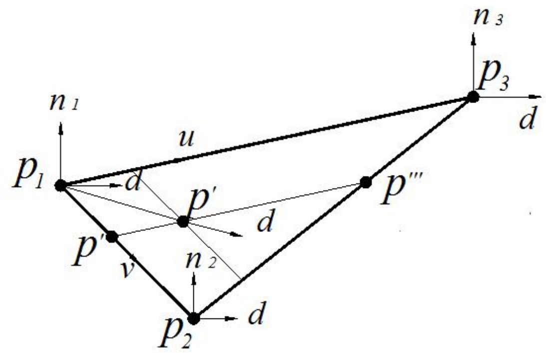
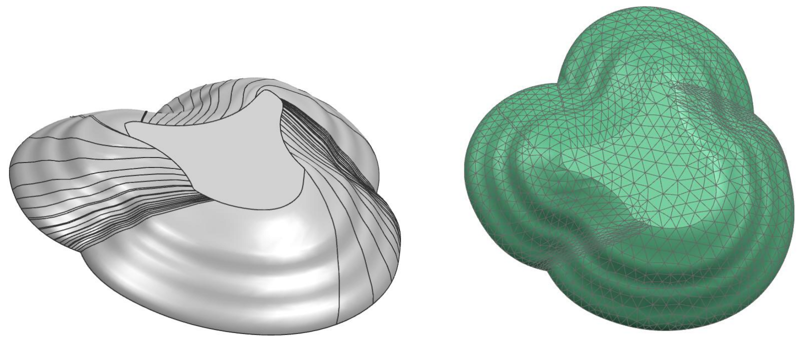


Figure 6. Calculation diagram of normal curvature of contact point of cutter that is inside the grid cell

Through the calculation of normal vector and main curvature of contact points of cutter on grid cell in this section, it has laid the groundwork for planning the path for curved surface of grid cell. The diagram of model in this paper and the diagram of triangular grid model are shown in figure 7 (a) and (b):



(a) (b)

Figure 7. Diagram of model in this paper and triangular grid model

III. GENERATION OF PATH

In the generation of cutter-path, step length and row pitch are particularly important factors, they determine the value of consistent residual height and have great effect on the quality of machining and efficiency of machining [17]. After determining the boundary curve of the grid, the path of contact points of cutter is obtained by dispersing the boundary curve. On the curved surface of triangular, the row pitch of contact points of cutter between the adjacent cutter-path and the distance of contact points of cutter on the same cutter-path is determined by using the characteristics of contact points of cutter on triangular grid cell, consistent residual height and step length of error of arc height, then mapping the cutter contact points which are on the curved surface of grid onto the field of parameter and re-planning the mapping cutter contact points in parametric field, the contact points in parametric field are obtained. Use fitting algorithm of NURBS curve to fit the contact points of cutter onto spiral path in parameter filed and inverse mapping it onto curved surface of grid to obtain the corresponding spiral machining path.

a. Dispersing the biggest contour line into contact points of cutter by using constant step length of arc height

In this paper, the geometric step length which satisfying the requirement of machining precision is estimated by using the algorithm of constant step length of arc height and the the geometric characteristics of curved surface of grid [18], the current contact points of cutter is obtained by using algorithm of non-uniform recursive interpolation, using the constant step length of arc height and row pitch of machining to determine the position of the next contact point of cutter. This algorithm is simple, practical and highly efficient; it can obtain contact points of cutter along the direction of the cutter-path and meet the requirements on the path of cutter. Through this algorithm, the obtained clusters of annular biasing curve can be dispersed into a cloud of points which is orderly arranged one by one.

As is shown in figure 8, the curve is dispersed into contact points of cutter according to error of arc high. Firstly, the coordinate value of midpoint $P_{i+1/2}$ of arc P_iP_{i+1} in the field of interpolation and the coordinate value of $P'_{i+1/2}(u)$ is calculated, the distance between the point $P_{i+1/2}$ and the point $P'_{i+1/2}(u)$ is the arc height h (as is shown in figure 9). By comparing the arc height h with the error of arc height which is set previously, the calculation is ended if the arc height is between the range of error ($h<\delta$), otherwise the recursive calculation is called. Using this algorithm can not only guarantee the least obtained interpolation points, it can also satisfy the requirements of step length of arc height at the same time, thus can greatly reduce the feeding time of cutter. The expression of calculation for arc height is:

$$h = \frac{| y(u_{i+u/2}) - (y_i + y_{i+1}) / 2 | g \sqrt{(y_i - y_{i+1})^2 + (x_{i+1} - x_i)^2}}{| x_{i+1} - x_i |} \tag{7}$$

In the expression: $y(U_i)$ is the vector value of y axis of a point $P(U_i)$ on the curve of cutter-path, x_i, y_i, x_{i+1} and y_{i+1} are the coordinate value of $P(U_i)$.

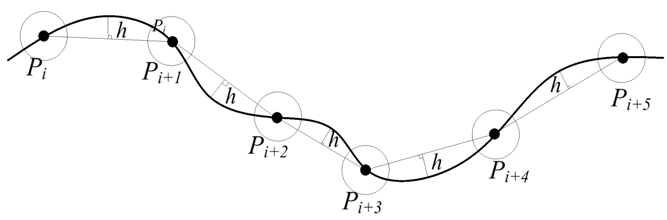


Figure 8. Diagram of dispersing the curve into contact points of cutter

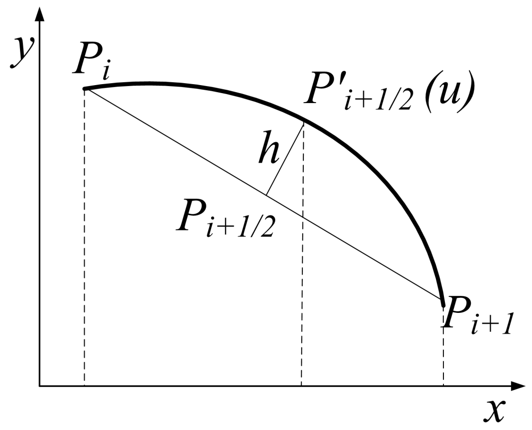


Figure 9. Diagram of the error of arc height

b. Determination of cutting row pitch of contact points of cutter between the adjacent cutter-path

The cutting row pitch of contact points of cutter between the adjacent cutter-path in parametric field is determined by horizontal distance of cutting-path in machining of grid. The row pitch directly affects the precision and efficiency in machining of curved surface, smaller cutting row pitch will lead to longer cutter-path and increase the amount of calculation, it has serious impact on efficiency of machining. The row pitch is determined by consistent residual height usually, determination of cutting row pitch is an important link under the condition of satisfying the quality of machining and efficiency of machining.

The geometric relationship between cutting row pitch L and the horizontal distance of contact point of cutter between the adjacent cutter-path l is expressed as follows:

$$L = l / cos\theta \tag{8}$$

In the expression: θ is the angle between the the horizontal plane and triangle plane of curved surface of triangular grid on which the contact point lies, and $cos\theta=|n\cdot k|$, in the expression: $\boldsymbol{n}$ is the unit normal vector of triangle plane, $\boldsymbol{k}$ is the unit normal vector of the X-Y plane, the normal vector of triangular grid is calculated by using weighted algorithm [13], so L can be expressed by [12]:

$$L = \frac{|\rho|\{4(r_e+\rho)^2(h+\rho)^2-[\rho^2+2r_e\rho+(h+\rho)^2]^2\}^{\frac{1}{2}}}{[(r_e+\rho)(h+\rho)]} \tag{9}$$

In the expression: h is residual height, r_e is the radius of cutter, ρ is the curvature of contact point of cutter on the curved surface of grid, it is calculated by using main curvature of vertex and main direction on cell with combination of Euler formula through the algorithm of quadratic weighting [14]: when the curved surface is convex, the value of ρ is positive, otherwise, the value of ρ is negative. Practice it is usually simplified to the expression:

$$L = \sqrt{\frac{8hr_e\rho}{\rho+r_e}} \tag{10}$$

With consideration of quality and efficiency of machining, the minimum value of row pitch is chosen as the biasing row pitch. Assuming that the number of contact points of cutter is n on the initial machining path, Li $(i = 1.2.3... n)$ is the biasing row pitch of cutter for each contact points of cutter, among them, L_{min} is the minimum row pitch in the row pitches with their number is n, it has ensured the consistent residual height and the minimum row pitch L_{min} can be expressed by:

$$L_{min} = Min(L_{i(}i - 1.2.3\ldots n)) \tag{11}$$

The biasing contact point of cutter $P_{i,\,j+1}$ is searched by using the tangent plane $S_{i,\,j}$ which is established by contact point of cutter on the prior cutter-path, assigning the contact point of cutter $P_{i,\,j}$ which is on the prior cutter-path as the initial search point to search the next biasing contact point of cutter. Based on contact point of cutter $P_{i,\,j}$, the position of $P_{i,\,j}$ can be divided into varieties of circumstances. The biasing contact point of cutter is the intersection of line l which is established on foundation of $P_{i,\,j}$ and the triangular grid cell, through biasing calculation of all the contact points of cutter on prior cutter-path, the biasing contact points of cutter on the next annular cutter-path is obtained.

By the calculation of row pitch of contact points of cutter, the solution of the contact points of cutter on the next machining path is shown in figure 10:

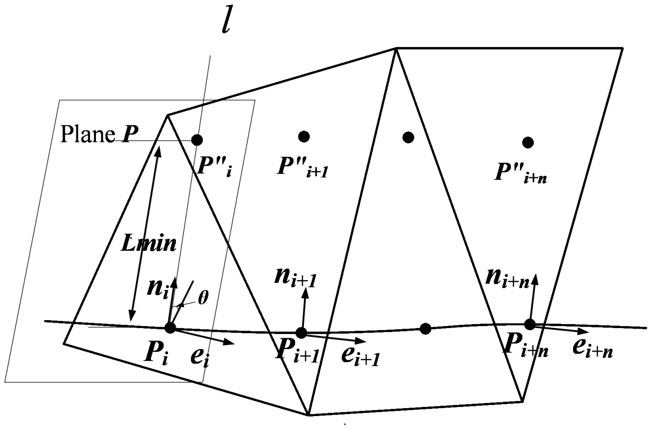


Figure 10. Solution of the contact points of cutter

$\boldsymbol{e_i}$ is the tangent vector of contact points of cutter P_i on machining path, $\boldsymbol{n_i}$ is the normal vector of contact points of cutter P_i on machining path, establishing a plane $\boldsymbol{P}$ on the point P_i with its normal vector is the $\boldsymbol{N_i}$ which is the normal vector of triangular grid cell, biasing the point P_i along the direction of normal vector $\boldsymbol{n_i}$, the biasing contact point of cutter P''_i is obtained which is contact point on the adjacent machining path.

The above algorithm can be used in machining of plane, the curvature and normal vector of contact point of cutter should also be taken into considered during the machining of curved plane, assuming the angle between the vector of $\boldsymbol{n_i}$ and plane $\boldsymbol{P}$ is θ, so the practical biasing row pitch of contact point of cutter on convex curved surface or concave curved surface should be $L' = L_{min}/cos\theta$. The obtained contact points of cutter on model are shown in figure 11.

Therefore, through the relationship expression between the error of arc height h and row pitch L, the annular contact points of cutter on curved surface can be obtained as are shown in figure 11, not only can this algorithm ensure the residual height of cutter-path is in the tolerance range with which the cutter has small change of curvature, but also it can make the residual height at the position of small corner meet the requirements.

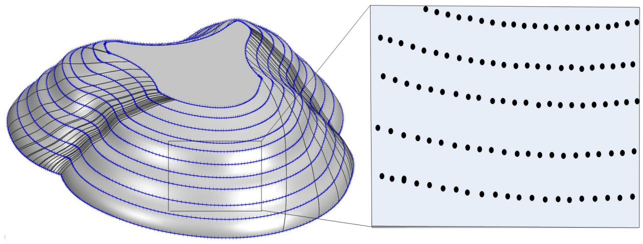
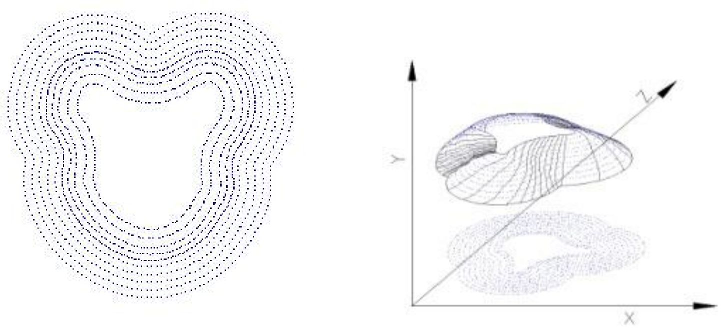


Figure 11. Biasing contact points of cutter on curved surface of grid

c. Mapping of contact points of cutter in parametric field and generation of cutter-path through inverse mapping

Through mapping the discrete contact points of cutter which are on the boundary of curved surface in parametric field, using algorithm of NURBS fitting [19] to interpolating and fitting the contact points of cutter. This algorithm can make the cutter-path consistent with the boundary of curved surface of grid, because of connection of spiral path, the redundancy of cutter-path is reducing, the continuity of cutter-path is improving at the same time.

The mapping rule between the parametric field and the original parametric field aiming at the problem of mapping, setting the parameter direction of $\boldsymbol{u}$ is the direction of X in original parametric field, the parameter direction of $\boldsymbol{v}$ is the direction of Y in original parametric field, projecting contact point of cutter on the cluster of annular curves on triangular grid on the plane of XY along the direction of -Z, the mapping points in parametric field are obtained. Through re-planning of mapping contact points of cutter in parametric field, the cutter-path is generated which is corresponding to practical machining path which is regenerated on triangular grid in practical machining. The projecting diagram of contact points of cutter which is on the cutter-path of curved surface of grid is shown in figure 12 (a).



(a) Projecting diagram of contact points of cutter (b) Diagram of mapping rules

Figure 12. Diagram of mapping rules of contact points of cutter

The obtained contact points of cutter is one-to-one corresponding with the practical contact points of cutter on the curved surface of triangular grid, as are shown in figure 12 (b). For any discrete contact points of cutter *(s, t)* in parametric field which is on cutter-path of curved surface of triangular grid along the direction of z, there is only one unique point *(u, v)* that is corresponding with it in the corresponding parametric field. The mapping rule between the practical contact point of cutter *(s, t)* and the contact point of cutter *(u, v)* in parametric field forming a one-to-one relationship of mapping.

As to inverse mapping, the equation of grid cell and the rules of inverse mapping are determined according to the parametric characteristics of grid cell on curved surface of grid (the direction of inverse mapping is opposite to mapping), mapping the contact points of cutter in parametric equation of NURBS according to the rules of mapping of, the spiral path on the curved surface of grid is obtained through solving the intersection on surface of grid cell.

d. Planning of spiral path in parametric field

CNC machine tools with function of NURBS curve have been developed in High-grade CNC system by manufacturers of CNC equipment, for machining of free-form curved surface with high precision and efficiency. The spiral path which is obtained by using traditional ways, using straight line to connect curve is far away from the requirement of machining, thus the way of connecting the adjacent path with transitional path is proposed. When the length of transition path between the two annular adjacent cutter-paths is larger, the rate of change of cutting force between the cutter and curved surface is smaller, so it reduces the vibration of cutter to the machine tool, protects the machine tool and improves the machining precision of curved surface at the same time. However, too large length of transition path will affect the efficiency of machining. Thus, use the geometric characteristics parameters of contact points of cutter on triangular grid surface--curvature, selecting the point with maximum curvature in annular contact points of cutter. Planning the contact points of cutter on transition path on this basis of this, the contact points of cutter on transition path in mapping field are obtained, it also avoids the problem of change rate of curvature is too large at the position with sharp angle.

Because the mapping rules between the contact points of cutter in parametric field *(u, v)* and the practical contact points of cutter on curved surface of triangular grid *(s, t)* is a one-to-one relationship, therefore, the contact points of cutter on the annular cutter-path in parametric field are numbered by $P_{i,j}$, in which $\boldsymbol{i}$ is the number of annular cutter-path, $\boldsymbol{j}$ is the number of contact points of cutter. The specific steps of generation of spiral path in mapping field are as follows:

Step1. Set the length of transitional path;

Step2. Start from contact points of cutter on annular boundary curve, selecting the contact points of cutter $P_{1,1}$ on curved surface of grid which has maximum curvature and is obtained by mapping the contact points of cutter *(u, v)* in parametric field onto corresponding points on curved surface of grid. Outward biasing from the point $P_{1,1}$ along the direction of its normal vector with a effective value of cutter's radius $\boldsymbol{r_e}$ to obtain the point on path $P'_{1,1}$, if they have the same curvature, the direction of normal vector of cutting point is used as standard. Biasing the adjacent point $P_{1,2}$ along the direction of its normal vector with the distance of $\boldsymbol{l_i}$. The expression of $\boldsymbol{l_i}$ is as follows:

$$l_i = \frac{l-i}{l} r_e \tag{12}$$

Taking the same steps to bias contact points of cutter on the same annular path, until the last contact points of cutter is overlap with any point of them, deleting the points $P_{1,1}$, $P_{1,2}$, $P_{1,3}$...$P_{1,n-2}$, $P_{1,n-1}$ and $P_{1,n}$, then the number of generated contact points of cutter is n, the other contact points of cutter on the other annular boundary curve are biasing according to the step 1 and step 2, thus, series of contact points of cutter with number is obtained, as are shown in figure 13.

Step 3. Fitting the generated contact points of cutter with interpolation curve, in this paper, using three times of algorithm of least squares fitting NURBS curve to fit the spiral curve:

$$s(t)=\sum_{k=0}^{3}P_k B_{k,3}(t), \qquad t\in[0,1] \tag{13}$$

In the expression: P_k is the point of control, $B_{k,3}(t)$ is the three times of basis function of b-spline.

Thus a continuous and smooth spiral curve that meets the requirements of machining of curved surface is obtained, as is shown in figure 14.

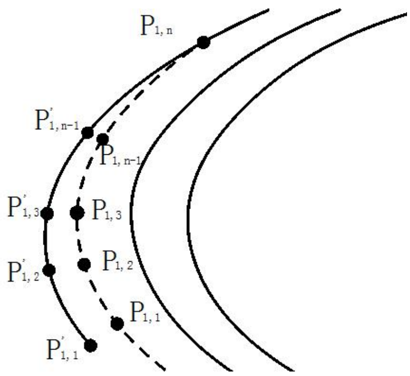


Figure 13. Planning of contact points of cutter on transition path

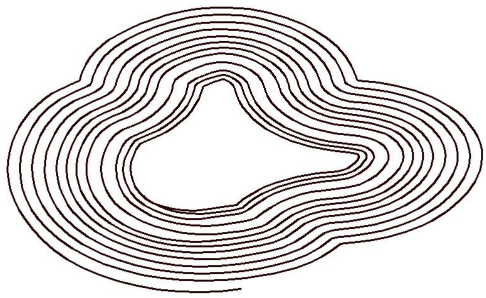


Figure 14. Diagram of fitting the contact points of cutter with interpolation NURBS curve

Step 4. Obtain the spiral path of contact points of cutter on curved surface of grid by inverse mapping the obtained spiral path in mapping field onto curved surface of grid.

In this section, through planning the mapping contact points of cutter, the spiral path in parametric field is obtained, and then inverse mapping the spiral path onto the curved surface of grid, finally the spiral machining path for contact points of cutter that meets the requirements of residual height and constant error of arc height is obtained. if the machining path for contact points of cutter is needed, only need to bias half value of cutter's radius.

IV. Inspection and solution of interference

When the concave area which is formed by triangular grid is smaller than the radius of cutter, the phenomenon of partial interference may be existed in that area and cause over cutting. Through the curvature of contact points of cutter on the triangular grid cell, the field of interference can be detected. The algorithm is as follows: projecting the position of head of ball-end cutter onto the surface of grid model to obtain a circular projection area, using the position relationships between triangle grid cell and ball-end cutter in the projection area to judge whether there is interference in that area. The relationships of them are shown in figure 15:

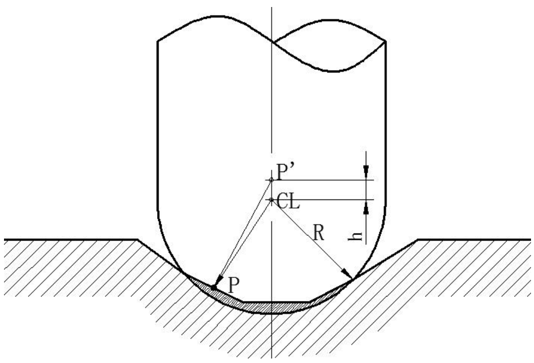


Figure 15. Diagram of interference in machining of triangular grid model

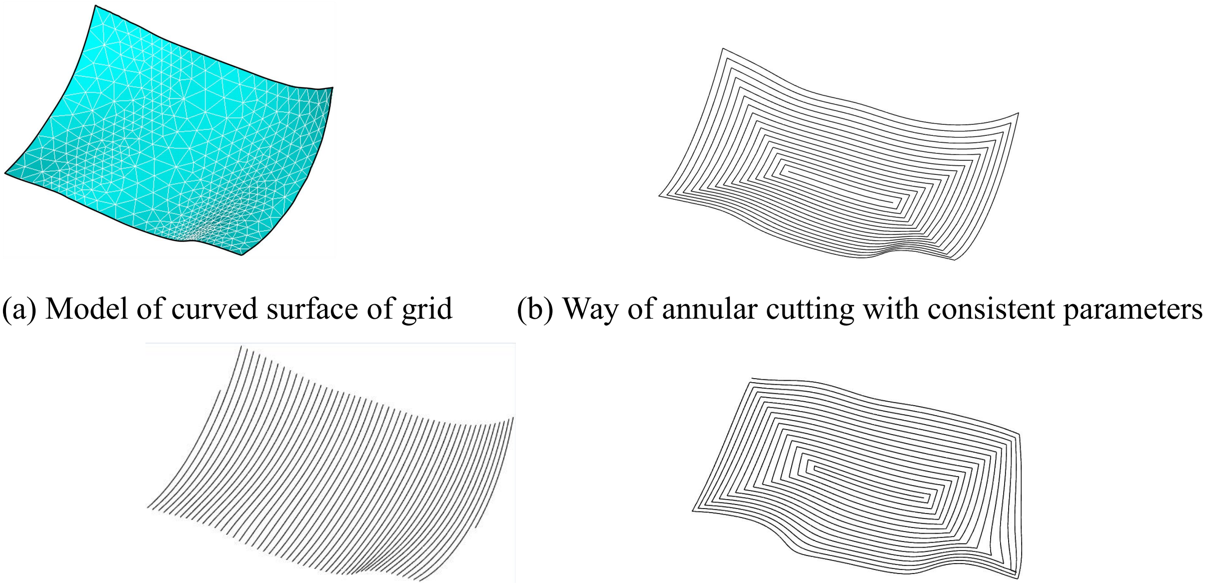
According to the above diagram, the judgment standard of whether there is interference in the area by the expression of geometrical relationships: $||CL - P|| < R$, In the expression: CL is the contact point of cutter, P is the interference point of cutter and grid cell, R is the radius of cutter, P' is the intersection point between the normal vector of contact point of cutter P and the axis of cutter. In order to solve the interference, this paper upward moving the cutter to a certain height $\boldsymbol{h}$ to avoid interference, In the diagram, $\boldsymbol{h}$ is the subtraction of lengths of projection of line PP' h and line PCL, also it can meet the machining requirements of consistent residual height.

V. ANALYSIS OF EXAMPLES

To verify the feasibility of algorithm in this paper, using the curved surface model of triangular grid as is shown in figure 16 (a), using *MATLAB* to read the model of STL grid for programming and planning the path for it. Verify the superiority of the machining path of triangular mesh model which is generated by algorithm in this paper by comparing it with the machining paths which is generated by way of annular cutting with consistent parameters and cross sections respectively.

To simulate the model of curved surface of grid as is shown in figure 16 (a), the radius of selected cutter is $R = 10\ mm$, the residual height is $h = 0.1\ mm$, the error of arc height is $\delta = 0.025\ mm$. The machining path which is planned by way of annular cutting with consistent parameters is shown in figure 16 (b), it is mainly biasing the boundary curve and cutter lifting is needed before start cutting the next annular path after finished the current path. It is easily causing the vibration and impact of the machine tool, the efficiency is relatively low. The machining path which is planned by way of cross sections is shown in figure 16 (c), the machining path is short and needed continuous cutter lifting, feeding or relieving of cutter. The fluctuation of load is large and cause vibration and impact of machine tool. The machining path which is planned by algorithm in this paper is shown in Figure 16 (d), it can generate a continuous spiral machining path that can reduce the number of feeding or relieving of cutter, enhance the machining efficiency, Because of the smooth machining path, it can reduce the vibration and impact of the machine tool and ensure the quality of machining indirectly.

The results of simulation show that: The connections of discontinuous parts in machining path which is generated by way of annular cutting with consistent parameters and way of cross sections respectively must be achieved by cutter lifting; sudden changes of directions of cutter during machining will impact the machine tool. Compared with the traditional ways, the algorithm in this paper can generate a smooth and continuous path with fewer changes of velocity direction, it can effectively reduce the numbers of cutter lifting which are caused by the discontinuities of path, effectively restrain the fluctuation of load of cutter, reduce the wear of cutter, realize the continuous feeding of cutter and is suitable for high speed machining.



(a) Model of curved surface of grid (b) Way of annular cutting with consistent parameters

(c) Way of cross section (d) Algorithm is this paper

Figure 16. Diagram of model of curved surface of grid and machining path generated by three ways

Diagram of comparison between maximum residual height vs. radius of cutter which is generated by annular cutting and algorithm respectively are shown in figure 17, the maximum residual height is obtained by measuring and calculating path of simulation, the area of the model is selected previously. The phenomenon's of over cutting and under cutting are often existed during simulation through the way of annular cutting, because this way has not taken fully account of characteristics of contact points of cutter which are on the curved surface of grid. Using different radius of cutter to simulate calculation, the maximum residual height is obviously increasing with the increase of radius of cutter through the way of annular cutting. The increase of maximum residual height through the algorithm in this paper is obviously smaller compared to that through the way of annular cutting, thus it indirectly reflects the quality of the machined surface.

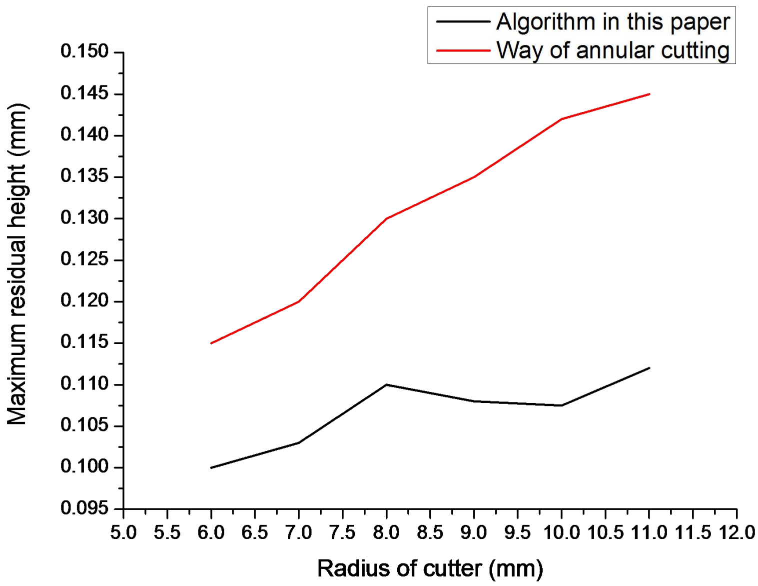


Figure 17. Diagram of comparison between maximum residual height vs radius of cutter

VI. CONCLUSIONS

In this paper, based on the grid of STL, full considering the characteristics of grid cell and proposing an algorithm of planning of spiral machining path with consistent residual height. It makes more precise calculation by using the algorithm of shape-area weight to calculate the normal vectors of contact points of cutter and taking the effects of the area and shapes of the triangles to normal vectors into account. The more precise consistent residual height is calculated by combining the main curvature and main direction of contact points of cutter with Euler formula by using algorithm of quadratic weighting, and then the row pitch is determined by the obtained consistent residual height and error of arc height. Because the contact points of cutter are operated separately thus ensuring the intersection of adjacent path will not exist, at the same time, the interference during machining can be detected and eliminated timely through the obtained information of curvature. Using algorithm in this paper, a superior spiral machining path is generated with advantages of consistence boundary, smooth and continuous path and without repeatedly need for feeding or relieving of cutter.

REFERENCES

[1] Zuqiang Long, Wen Long, Yan Yuan, Xiaobo Yi, "Design of Backing-up Fuzzy Controllers Based on Variable Universe of Discourse", International Journal on Smart Sensing and Intelligent Systems, Vol.6, No.2, 2013, pp. 505-522.

[2] Zuqiang Long, Yan Yuan, Yuebing Xu, Shehui Du, "High-Accuracy Positioning of Lathe Servo System Using Fuzzy Controllers Based on Variable Universe of Discourse", International Journal on Smart Sensing and Intelligent Systems, Vol.7, No.3, 2014, pp.1114-1133.

[3] SUN Yuwen, GUO Dongming, JIA Zhenyuan, "Iso-parametric tool path generation from triangular meshes for free-form surface machining", The International Journal of Advanced Manufacturing Technology, Vol.28, No. 7-8, 2006,pp.721-726.

[4] REB Y F, YAN H T, LEE Y S., "Clean-up tool path Generation by contraction tool method for machining complex polyhedral models", Computers in industry, Vol.54, No.1, 2004, pp. 17-33.

[5] SYRESH K, YANG DCH, "Constant scallop-height machining of free-form surface", Journal of Engineering for Gas Turbines and Power, Vol.116, 1994, pp. 253-259.

[6] AGRAWAL R K, PRAYIHAR D K, CHOUDHURY A R, "Optimization of CNC isoscallop free form surface machining using a genetic algorithm", International Journal of Machine Tools & Manufacture,Vol.46, No. 7-8, 2006, pp.811-819.

[7] ZHU Hu, LIU Zhijun, FU Jianhui, "Spiral tool-path generation with constant scallop height for sheet metal CNC incremental forming", Advanced Manufacturing Technology Vol.54, No. 9-12, 2011, pp. 911-919.

[8] LEE E., "Contour offset approach to spiral tool-path generation with constant scallop height", Computer-Aided Design, Vol.35, No.6, 2003, pp. 511-518.

[9] XU Jinting, LIU Weijun, DIAN Hongyou, WANG Huabing, "Constant Scallop Tool Path for Triangular Surface Machining", China Mechanical Engineering, Vol.11, 2010, pp. 193-198.

[10] ZHOU Bo, ZHAO Jibin, LIU Weijun, LI Lun, "Generation method for five-axis NC spiral tool path based on parametric surface mapping", Journal of Systems Science &

Complexity, Vol. 05, 2013, pp.676-694.

[11] PATELOUP.DUCE , RAY P.B-spline approximation of circle arc and straight line for pocket maching. Computer-Aided Design, Vol. 42, No. 9, 2010, pp.817-827.

[12] Li M, ZhANG L C, MO J H., "Tool-path generation for sheet metal incremental forming based on STL model with defects", International Journal of Advanced Manufacturing Technology, Vol. 63, No. 5-8, 2012,pp. 535–547.

[13] LU B, CHEN J, Ou H. Feature-based tool path generation approach for incremental sheet forming process, Journal of Materials Processing Technology, Vol. 213, No.7, 2013, pp.1221-1233.

[14] SHEN Huicun, LI Jianhua, ZHOU Laishui, "Estimation of Triangular Mesh Vertex Normal Vector and Discrete Curvature", Computer Engineering and Application, Vol.26, 2005, pp. 12-15.

[15] HU Jinming, YANG Jingxu, ZHOU Yuansheng, "A weighted Compensation Algorithm for Normal Vector of Cutter Contact Point Ob triangular Mesh Surface", China Mechanical Engineering, Vol.10, 2012, pp.1226-1229.

[16] DONG Chenshi, WANG Guozhao, "Curvatures estimation on triangular mesh", Journal of Zhejiang university Science, Vol.5, No.2, 2004, pp. 218-221.

[17] CHEN X, Schmitt F., "Intrinsic surface properties from surface triangulation", Proceedings of the European Conference on Computer Version, 1992, pp. 739-743.

[18] JI Junfeng, Zhou Laishui, An Luling, ZHUANG Weinan, "On Toolpath Planning for NC Maching of a free form Blade surface", Mechanical Science and Technology for aerospace engineering, Vol. 05, 2008, pp. 652-656.

[19] ZHAO Peng, Lou Pei huang, LIU Mingdeng, HU Ronghua, "Research on the NURBS Curve Fitting for Tool Path Generation", Computer Integrated Manufacturing Systems, Vol. 07, 2011, pp.1454-1459.